



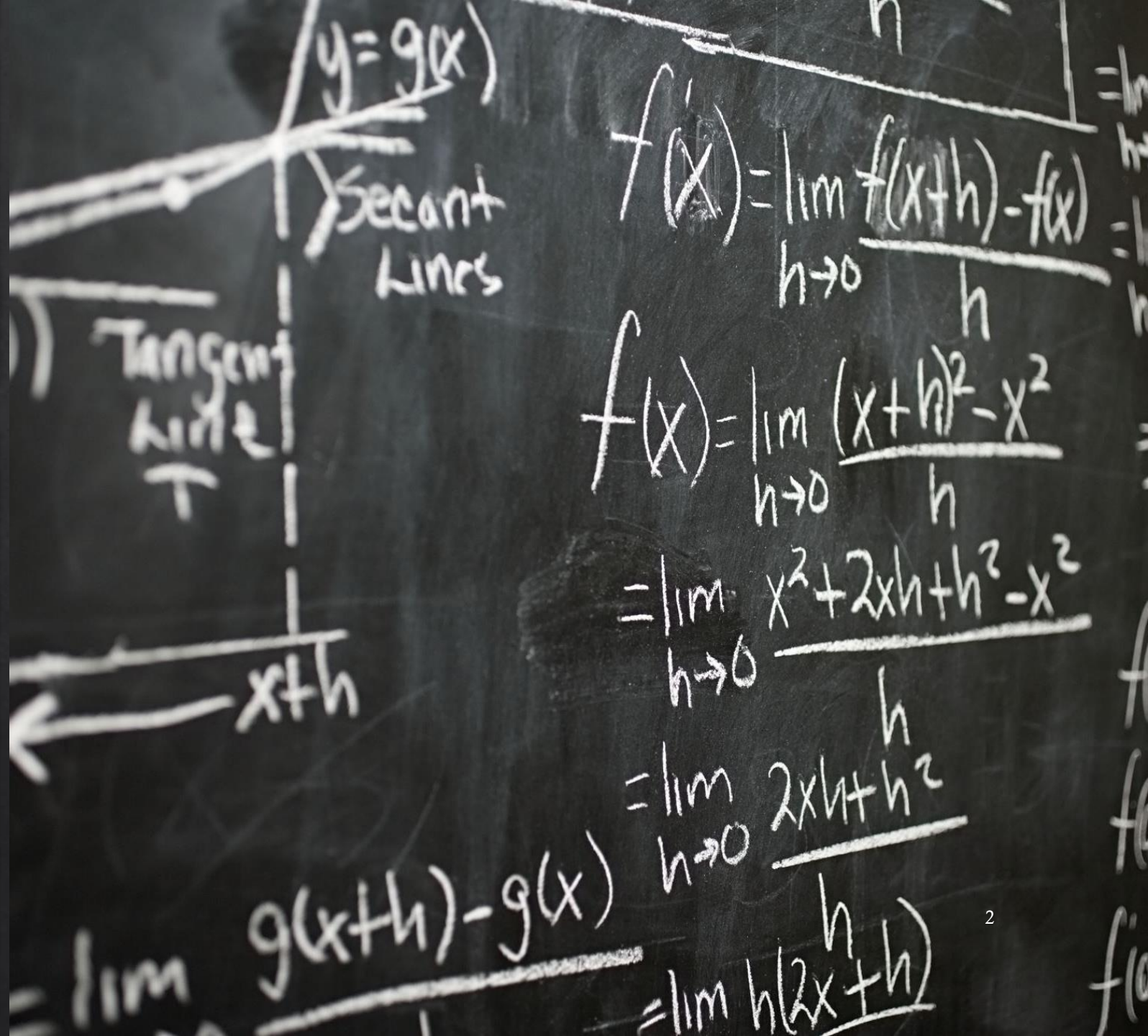
Patrick Gaule is an applied microeconomist who works at the intersection of labor economics and the economics of science and innovation. He is currently an associate professor at the University of Bristol. He holds a PhD from the Ecole Polytechnique Federale de Lausanne in Switzerland and was a postdoctoral research fellow at the MIT Sloan School of Management, the National Bureau of Economic Research, and Harvard University. Patrick Gaule joined IZA as a Research Affiliate in March 2016

How to encourage and develop mathematically gifted children

Patrick Gaule

University of Bristol &

Global Talent Lab



Introduction

- ◇ “Not everybody is equal in front of math” – my middle school math school-teacher
- ◇ Simple intuition that some kids find easier to learn mathematics
- ◇ One consequence, if your child has a talent for mathematics, (s)he may be able to learn mathematics at a more advanced level than other kids:
 - ◇ Should (s)he?
 - ◇ How?

Being good at math – a gift

- ◇ If you are good at math, you can get substantial returns on the labour market from it
- ◇ That is because pathways to many well-paid professions involve mathematics:
 - ◇ Computer science
 - ◇ Engineering
 - ◇ Economics
 - ◇ ...
- ◇ Even medicine, psychology, and law involve mathematics to some degree

Being good at math – and a burden?

- ◆ For diverse reasons, kids who are talented in mathematics may be more prone to difficulties socializing and/or be more fragile psychologically
- ◆ Being good at math (and arguably academic excellence) is often not perceived to be cool among kids in Western societies
- ◆ Talented kids may be in school with older, more mature kids
- ◆ Not uncommon for children on the autism spectrum to excel at mathematics

Outline



SHOULD YOUR
TALENTED KID LEARN
MORE MATH?



IF SO, HOW?

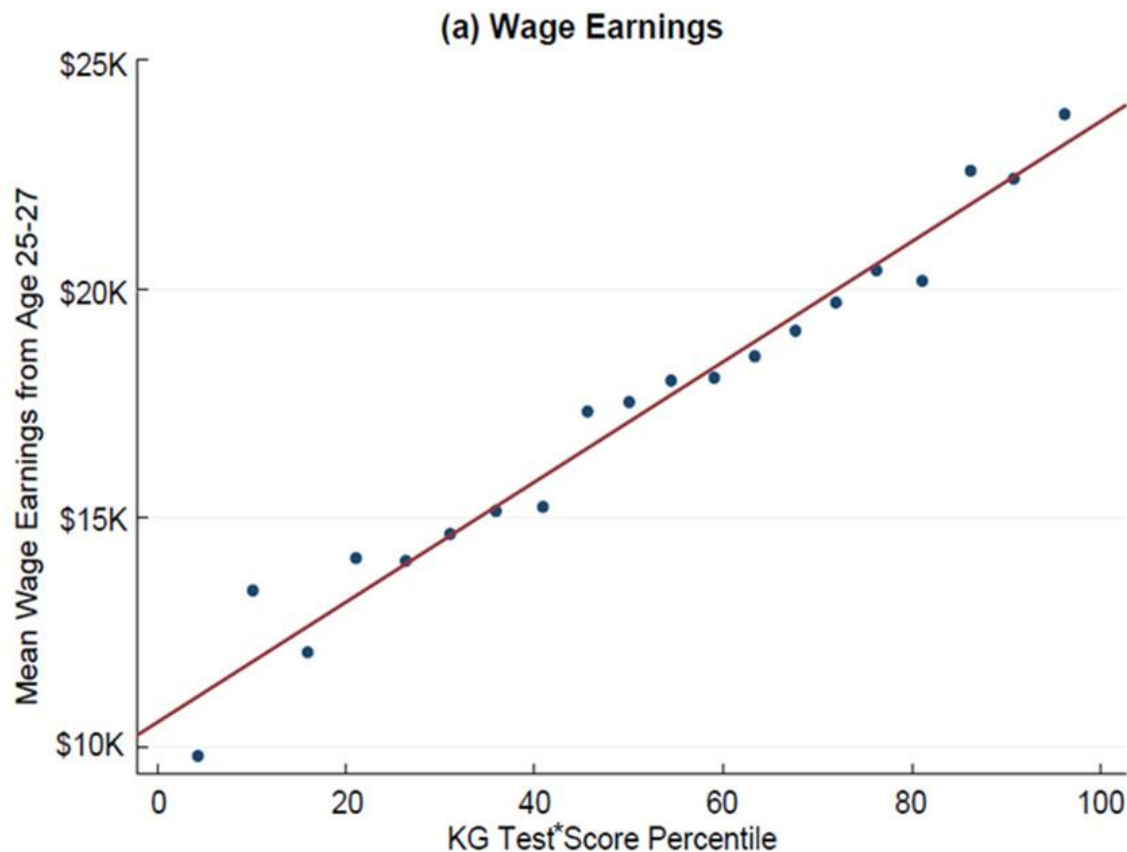


PRELIMINARY RESULTS
OF A RANDOMIZED
EXPERIMENT

Math and earnings (1)

- ◇ Kindergarten test scores (math + reading) correlate with adult earnings

Correlation of Kindergarten Achievement Score Percentiles and Wage Income at Ages 25–27



*Stanford achievement test, mean of verbal and math scores.

Chetty, Raj, et al. "How does your kindergarten classroom affect your earnings? Evidence from Project STAR." No. w16381. *National Bureau of Economic Research*, 2010.

Math and earnings (2)

- ◇ Data from the British Cohort Study
- ◇ A one standard deviation in math test score at age 10 is associated with a 14% increase in wages
- ◇ Source: Crawford & Cribbs

Specification	(1)	(2)	(3)	(4)	Number of observations
Controlling for	Reading and maths scores				
<i>Dependent variable:</i> <i>log earnings at age 30</i>					6,029
Std reading test score at age 10	0.061*** (0.010)	0.048*** (0.010)	0.044*** (0.010)	0.019* (0.010)	
Std maths test score at age 10	0.144*** (0.010)	0.119*** (0.011)	0.105*** (0.011)	0.073*** (0.010)	

Notes: *** denotes that the effect is significantly different from zero at the 1% level, ** at the 5% level, * at the 10% level. Standard errors are robust to heteroskedasticity. Specification 1 includes controls for sex and region. Specification 2 adds parents' income and education. Specification 3 adds ethnicity, housing tenure during childhood, father's occupational class, father's and mother's employment status at age 10, number of older and younger siblings at age 10, whether they are a twin, whether either parent had a major illness during the individual's childhood, mother's age at the individual's birth, mother's marital status at birth, the individual's birth weight, whether their mother drank during pregnancy, whether their mother smoked during pregnancy, whether they were breastfed and their birth weight. Specification 4 adds controls for the highest qualification attained.

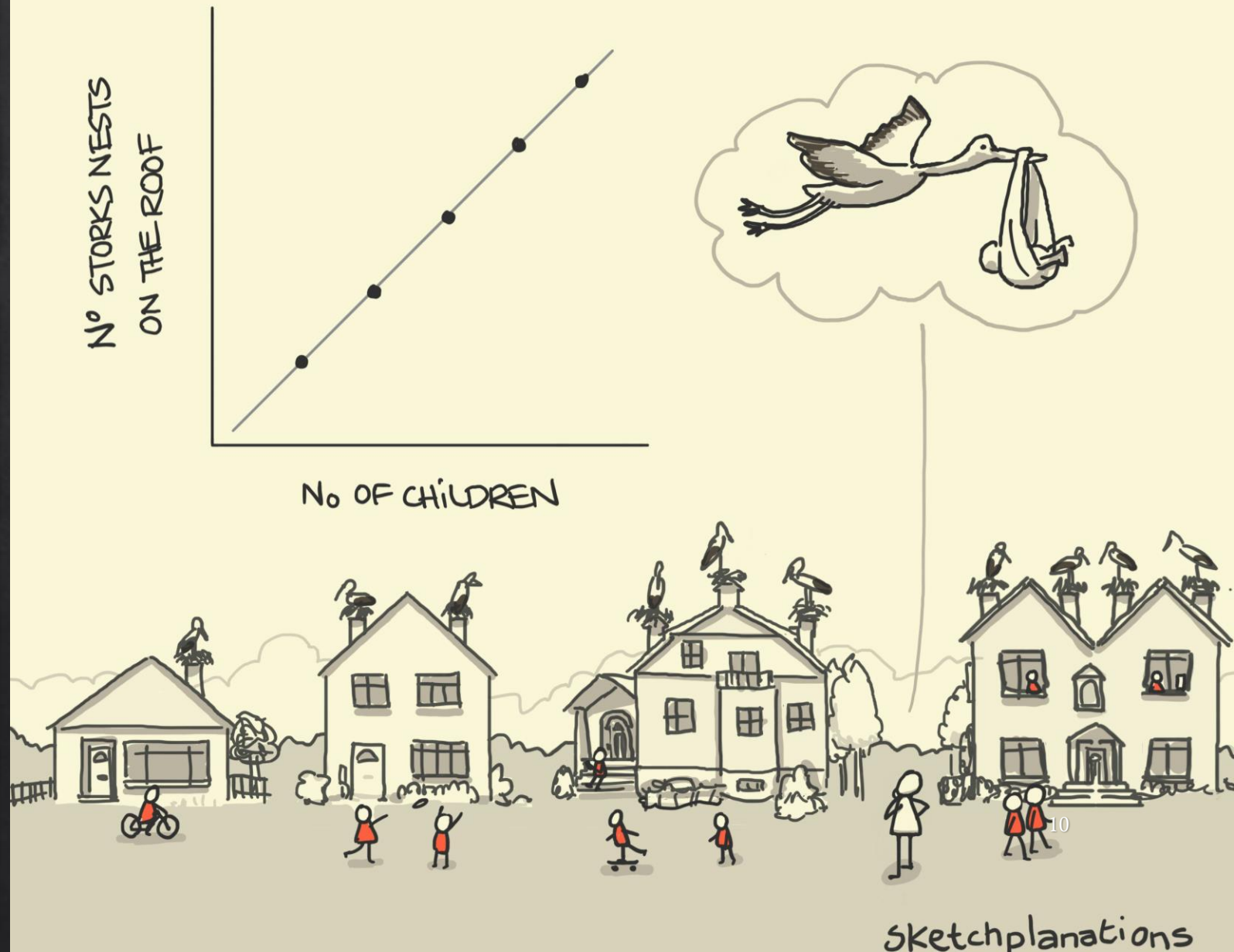
Math and earnings (3)

- ◆ Third-grade math tests are more strongly associated (than English score) with being in the top 1 percent of the income distribution
- ◆ A one standard deviation increase in 3rd grade math score is associated with a doubling in the likelihood of being in the top 1 percent of the income distribution
- ◆ Source: Appendix Table 1 in Bell et al. (2019)

Dependent Variable:	In Top 1% of Income Distribn. (per 1,000 Individuals)		
	(6)	(7)	(8)
3rd Grade Math Score (SD)	11.07*** (0.36)		8.08*** (0.46)
3rd Grade English Score (SD)		10.19*** (0.34)	4.15*** (0.42)
Fixed Effects	None	None	None
Mean of Dependent Variable	9.68	9.72	9.84
Observations	165,422	161,275	158,016

CORRELATION IS NOT CAUSATION

- ◇ We have seen that math test scores in school age are associated with adult earnings
- ◇ But this does not imply that it is because they were better at mathematics that they have higher adult earnings
- ◇ What one might to know is whether teaching more mathematics to a kid improves his/her adult earnings
- ◇ Here the evidence is less clear



Labor market returns to learning more math

- ◆ Fundamental difficulty: students who are keen to learn more mathematics are different from other students
- ◆ Goodman (2012) leverages changes in in minimum high school math requirements across states.
- ◆ He finds being forced to take an additional math courses increases earning by around 10% for black student but white students are not affected.
- ◆ Some other studies find similar results (positive effects of learning more maths, but not necessarily for all groups)



So, should your talented kid learn more math?

- ◆ There are good reasons to think that learning more math will enhance a talented kid's career prospects
- ◆ I also believe that participating in academic enrichment activities can help a kid find like-minded peers and improve his/her psychological well-being

Outline



SHOULD YOUR
TALENTED KID LEARN
MORE MATH?



IF SO, HOW?



PRELIMINARY RESULTS
OF A RANDOMIZED
EXPERIMENT

Different ways of learning (more) advanced math material

	Full time	Regular but part-time	One-off
In Person	Specialized schools, Grade skipping	Math circles, tutoring, extra classes	Camp
Virtual	Remote instruction	Art of Problem Solving	Virtual workshop

In person:
Better for socialization and
motivation

But also more expensive
and less suitable for those
who live in remote areas

Grade skipping

- ◆ Grade skipping is a form of academic acceleration enabling the student to skip entirely the curriculum of one or more years of school.
- ◆ Terrence Tao, now one of the world's most prominent mathematicians, GRADUATED from college at age 16 (and got his PhD at the age of 21)
- ◆ Grade skipping immediately exposes the student to more advanced material (otherwise designed for older students)



Terrence Tao aged 10, working with legendary mathematician Paul Erdos

Grade skipping

Advantages:

- ◇ Easy to implement
- ◇ Inexpensive for either the parents or the state

Disadvantages:

- ◇ Leaving friends behind, socialization with older kids may be difficult
- ◇ Potential gaps in knowledge since a grade has been skipped
- ◇ The student may not be ready to skip a grade in all subject

Empirical evidence on grade skipping

- ◆ Time-saving theory: grade skipping allows to finish training sooner and allows more time for creative productive
- ◆ A study of 3000+ mathematically precocious kids found that grade skippers are more likely to get PhD, produce more patents, and highly cited publications by age 50 than academically comparable non-skippers (Park, Lubinsky and Penbow 2013)
- ◆ Despite the intuitive concerns about the effect of grade skipping on well-being, the evidence from the psychology literature is mixed

Specialized schools

- ◇ Some countries have more elite schools with selective entry requirements
- ◇ Kids in these schools also tend to have academically stronger peers
- ◇ They may follow a more advanced curriculum
- ◇ High fees can exclude middle-class (let alone working class) families
- ◇ Students from remote areas may have to stay at their school and see the parents less



St Paul school

- ◇ Small classrooms (12-15)
- ◇ Highly-qualified and attentive staff
- ◇ Academic enrichment activities
- ◇ Private with high fees

Does Going to a Better School Improve Academic Performance?

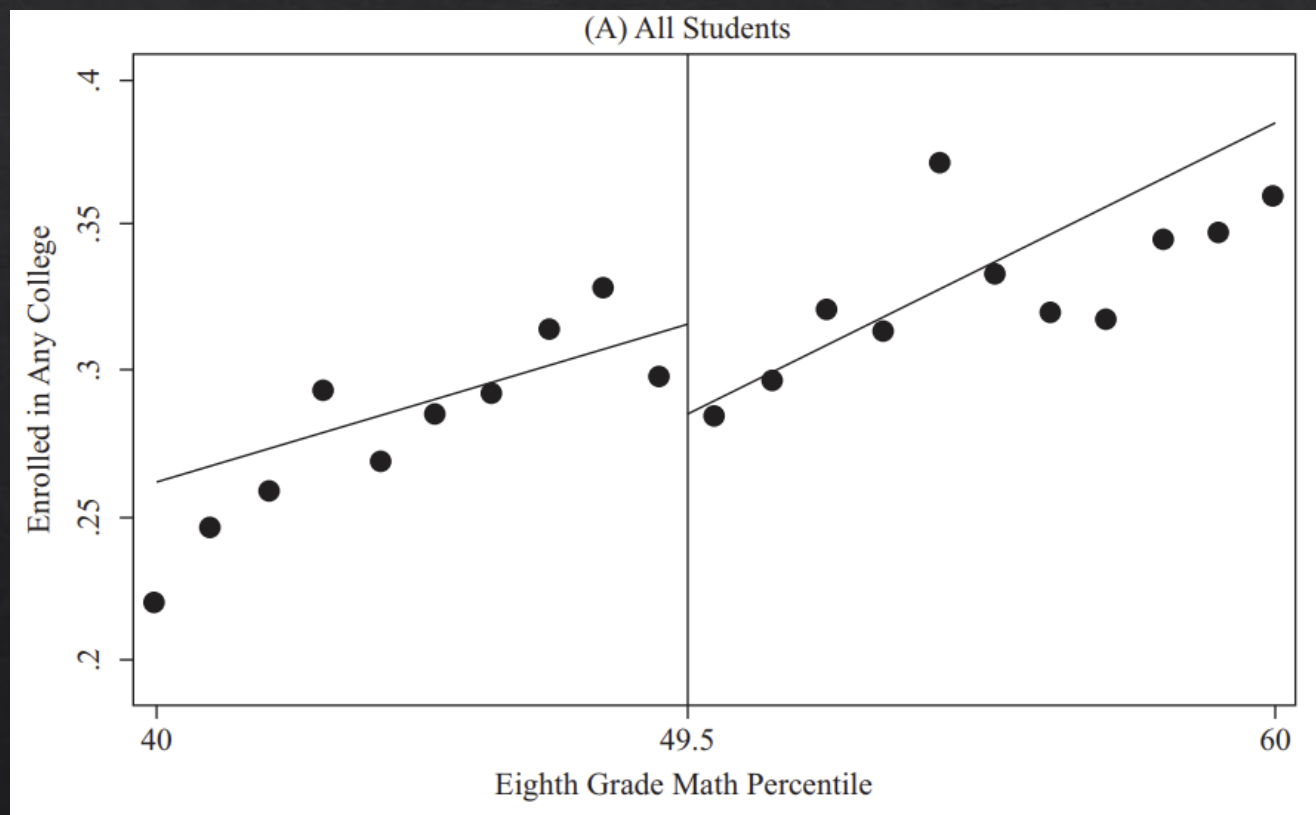
- ◆ Abdulkadiroğlu, Angrist, Pathak (2014) find no effect of attending high-achievement schools in Boston and New York
- ◆ Pop-Eleches and Urquolia (2012) find that going to a high performing school leads to better performance in a high-stake graduation test in Romania
 - ◆ children who make it into more selective schools realize they are relatively weaker and feel marginalized
 - ◆ parents reduce effort when their children attend a better school.
- ◆ But High-achievement class (within a school) where talented kids are sorted into lead to own significant gains in reading and math, concentrated among lower-income and black and Hispanic students (Card & Giuliano 2014)

Math circles

- ◇ These are out of school activities with weekly (or more) in person meetings
- ◇ Students typically work on solving problems under the guidance of an instructor
- ◇ Could be light-touch (1 hour) per week) or a lot of work indeed (20 hours per week)
- ◇ Can be problematic to attend for kids from remote areas
- ◇ Also an occasion for kids to socialize with like-minded peers
- ◇ No systematic evidence on effectiveness

Extra classes: A “double dose” of algebra

- ◇ Policy implemented by the public schools in Chicago
- ◇ Students scoring below the national median on a 8th grade math test receive were subsequently assigned to two periods of freshman algebra rather than the usual one period
- ◇ Cortes, Goodman and Nomi (2015) find that the double dose increased grades, high school completion and college enrollment
- ◇ Not really about talent kids



Tutoring

- ◇ Tutoring involves a qualified individual providing individual or small-group teaching outside normal schools, typically in exchange for monetary compensation
- ◇ Highly prevalent in some countries
- ◇ Can be delivered either in person or online
- ◇ Tutors normally have to be compensated so this may not be available to all kids
- ◇ The UK mathematics trust runs an online tutoring program whereby small groups of talented but disadvantaged kids are tutored by school-teachers
- ◇ Limited evidence on effectiveness

Camps

- ◇ Camps are in person-activities where kids get together for intensive training, typically outside the normal school year
- ◇ Top performers in mathematics competitions often go through camps
- ◇ Such camps are typically free to the students but selective in who gets in
- ◇ In the US, there are a lot of math camps offered by private providers
- ◇ PROMYS is a famous highly-selective 6-weeks camp; free for students whose parents are not rich
- ◇ Limited evidence on effectiveness

Wrapping Up So Far

- ◇ It is clear that talented kids can learn more math when offered the opportunity, and perhaps make friends in the process
- ◇ Many different mathematics enrichment activities are potentially available
- ◇ Not a lot of systematic evidence on relative effectiveness
- ◇ Some activities are expensive risk excluding talented kids with less ability to pay
- ◇ In-person activities tend to be held in cities and risk excluding talented kid from remote areas

Outline



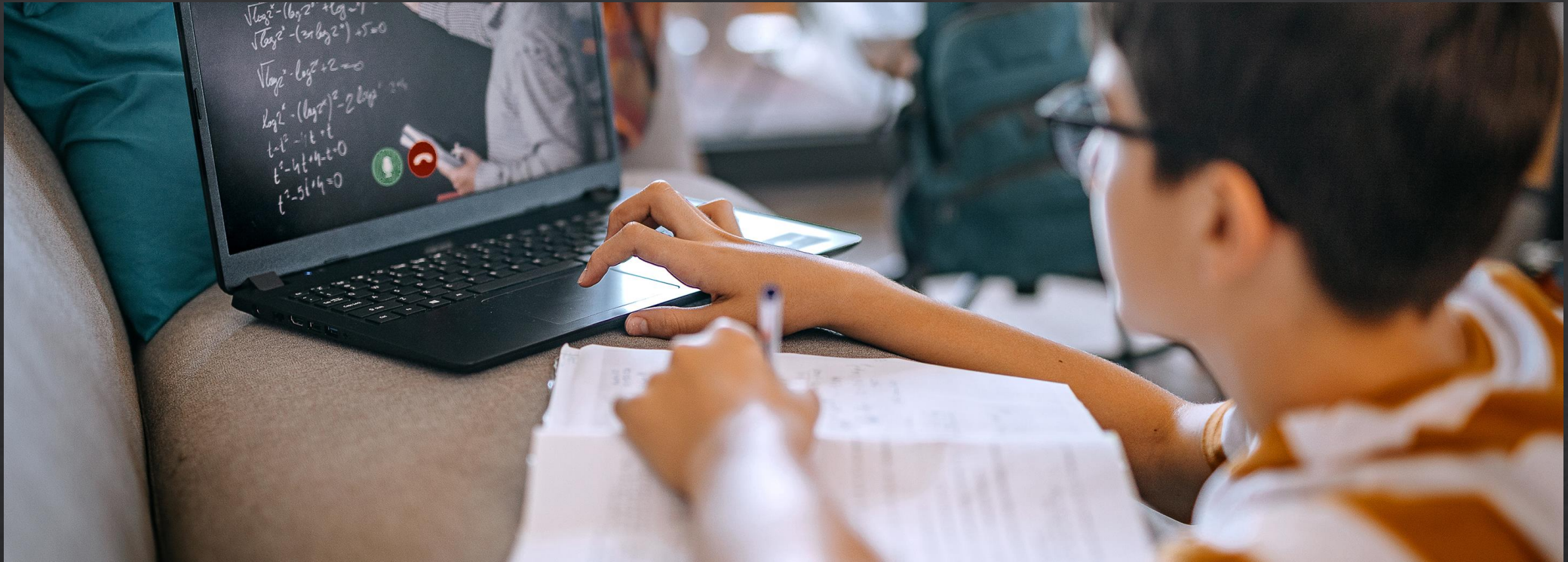
SHOULD YOUR
TALENTED KID LEARN
MORE MATH?



IF SO, HOW?



PRELIMINARY RESULTS
OF A RANDOMIZED
EXPERIMENT



Can Virtual Training Help Nurture Talent?

Virtual Training / Online Courses

- ◇ There are many online courses available
- ◇ Virtual courses have challenges in terms of student engagement, and don't (really) enable socialization between the students
- ◇ But they can be access from anywhere, you do not need to be in a city (or even a country) where there are good teachers
- ◇ In principle, they can be delivered at a lower cost than in-person courses

Art of Problem Solving

- ◆ Art of Problem Solving is a leading commercial provider of books and online courses
- ◆ They specifically targeted talented kids, and providing challenging material
- ◆ They focus on interactive problem-solving and rely on text-based interface
- ◆ In the U.S., many kids do their courses, including all of the top performers at the USA Mathematical Olympiad, they have a strong American focus
- ◆ Courses typically last 4 months and costs 400 to 600 USD to enroll



The STAR program

- ◇ Run by Global Talent Network
- ◇ Seeks to:
 - ◇ Identify very talented kids (STAR=Search for Talented Ramanujans)
 - ◇ Recognize and encourage them (including with cash prizes)
 - ◇ Give them the chance to learn by accessing a top-quality course
- ◇ Participants enter a lottery: half of them receive access to a course from *Art of Problem Solving*, the other half receive a book instead

The intervention

- ◊ Delivered by Art of Problem Solving
- ◊ 18 weeks course including 90 mins live sessions and homework
- ◊ An advanced and challenging course on combinatorics
- ◊ Control group: book on the same topic

ONLINE SCHOOL > INTERMEDIATE COUNTING & PROBABILITY (3733)

Intermediate Counting & Probability (3733)

Sunday
Oct 1, 2023 - Feb 18, 2024
12:00 - 1:30 PM ET (9:00 - 10:30 AM PT)

Overview | My Goals | Homework | Textbook | Message Board | Report | Teacher

Announcements

Please Read:

- Accessing the Online Classroom
- AoPS Honor Code

For Instructors:

- Edit Transcripts!
- Let's Go the Crypt!
- Instructor Script Comments
- Teacher Tools (Enrollment, Feedback, etc.)

Next Class

Review of Counting and Probability Basics
Sun, Oct 1, 2023
12:00 - 1:30 PM ET (9:00 - 10:30 AM PT)

Schedule

Week	Date	Transcript	Topic	Readings	Progress
1	Oct 1		Review of Counting and Probability Basics	Chapters 1, 2	<div style="width: 10%;"></div>
2	Oct 8		Principle of Inclusion & Exclusion	Sections 3.1-3.5	<div style="width: 0%;"></div>
3	Oct 15		Advanced Inclusion & Exclusion	Sections 3.6, 3.7	<div style="width: 0%;"></div>
4	Oct 22		Constructive Counting	Sections 4.1-4.3	<div style="width: 0%;"></div>
5	Oct 29		One-to-one Correspondences	Sections 4.4-4.6	<div style="width: 0%;"></div>
6	Nov 5		One-to-one Correspondences Continued and Pigeonhole	Chapter 5	<div style="width: 0%;"></div>
7	Nov 12		Constructive Expectation	Chapter 6	<div style="width: 0%;"></div>
8	Nov 19		Distributions	Chapter 7	<div style="width: 0%;"></div>
9	Dec 3		Mathematical Induction and Fibonacci Numbers	Chapters 8, 9	<div style="width: 0%;"></div>
10	Dec 10		Recursion and Catalan Numbers	Chapter 10	<div style="width: 0%;"></div>
11	Dec 17		Conditional Probability	Chapter 11	<div style="width: 0%;"></div>
12	Jan 7		Combinatorial Identities	Chapter 12	<div style="width: 0%;"></div>
13	Jan 14		Events with States	Chapter 13	<div style="width: 0%;"></div>
14	Jan 21		Generating Functions, Week 1	Sections 14.1-14.4	<div style="width: 0%;"></div>
15	Jan 28		Generating Functions, Week 2	Sections 14.5, 14.6	<div style="width: 0%;"></div>
16	Feb 4		Graph Theory, Week 1	Sections 15.1-15.4	<div style="width: 0%;"></div>
17	Feb 11		Graph Theory, Week 2	Sections 15.5, 15.6	<div style="width: 0%;"></div>
18	Feb 18		Bonus Topics and Challenging Problems	Chapter 16	<div style="width: 0%;"></div>

Endpoints

- ◆ We invite students from both treatment and control groups to a final test to measure their learning
- ◆ Participation in the final test is incentivized – top third of participants receive USD 500 cash prizes + a number of small prizes available on a lottery basis
- ◆ Our main endpoint is performance in the final tests
- ◆ Secondary endpoints: performance in national Olympiad, whether they study abroad

RCT results

	(1)	(2)
	Score in Final Test (standardized)	
Treatment group (course)	0.257* (0.132)	0.206 (0.131)
Control group (book): Omitted		
Controls	No	Yes
Obs.	229	229
Mean of D.V.	0	0
R2	0.02	0.45
Standard errors in parentheses		
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$		

Results are noisy but point estimates suggest that being assigned to the treatment group increases performance by 0.2 standard deviation

Interpreting RCT results

- ◆ These results are preliminary as the experiment is ongoing and an extensive analysis has not yet been conducted
- ◆ The preliminary results do suggest that being offered access to a course leads to higher performance, although the effect is not very large
- ◆ Substantial variation in the extent to which kids engage with the course

Conclusion

- ◇ Learning mathematics is good for a talented child's career prospects
- ◇ But also, by going to mathematics enrichment activities, a talented child may find and socialize with like-minded peer
- ◇ There are many ways to learn more advanced mathematics; but much remains to be learned about their relative effectiveness
- ◇ There is a role for the state or for philanthropy to ensure that talented kids from disadvantaged backgrounds and/or rural regions are not left behind
- ◇ Preliminary RCT results show that virtual courses can facilitate learning

References

- ◆ Chetty R., Friedman J. N., Rockoff J. E. (2014). Measuring the impacts of teachers II: Teacher value-added and student outcomes in adulthood. *American Economic Review*, 104(9), 2633–2679.
- ◆ Bell, A., Chetty, R., Jaravel, X., Petkova, N., & Van Reenen, J. (2019). Who becomes an inventor in America? The importance of exposure to innovation. *The Quarterly Journal of Economics*, 134(2), 647-713.
- ◆ Crawford, C., & Cribb, J. (2013). Reading and maths skills at age 10 and earnings in later life: a brief analysis using the British Cohort Study.
- ◆ Goodman, J. S. (2012). The labor of division: Returns to compulsory math coursework. HKS Faculty Research Working Paper Series.
- ◆ Park, G., Lubinski, D., & Benbow, C. P. (2013). When less is more: Effects of grade skipping on adult STEM accomplishments among mathematically precocious youth. *Journal of Educational Psychology*

References (continued)

- ◆ Card, D., & Giuliano, L. (2014). *Does gifted education work? For which students?* (No. w20453). National Bureau of economic research.
- ◆ Abdulkadiroğlu, A., Angrist, J., & Pathak, P. (2014). The elite illusion: Achievement effects at Boston and New York exam schools. *Econometrica*, 82(1), 137-196
- ◆ Pop-Eleches, C., & Urquiola, M. (2013). Going to a better school: Effects and behavioral responses. *American Economic Review*, 103(4), 1289-1324.
- ◆ Cortes, K. E., Goodman, J. S., & Nomi, T. (2015). Intensive math instruction and educational attainment: Long-run impacts of double-dose algebra. *Journal of Human Resources*, 50(1), 108-158.